



## Questions

### (1) Complete:

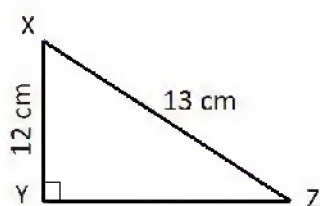
- 1)  $\tan 45 \sin 30 = \dots\dots\dots$
- 2) If  $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$  and the slope of  $\overleftrightarrow{AB} = \frac{1}{2}$  then the slope of  $\overleftrightarrow{CD} = \dots\dots\dots$
- 3) The line segment which is drawn between the two points (0 , 0) and (6 , 8) =  $\dots\dots\dots$  length unit.
- 4) If  $\sin 30 = \cos B$  then  $m (\angle B) = \dots\dots\dots$
- 5) If  $\sin A = \cos A$  then  $m (\angle A) = \dots\dots\dots$
- 6) The straight line whose equation  $y = 2x - 6$  its slope =  $\dots\dots\dots$   
and its intercepts from the y-axis a part of length  $\dots\dots\dots$  unit.
- 7) If  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$  and the slope of  $\overleftrightarrow{AB} = \frac{2}{3}$  then the slope of  $\overleftrightarrow{CD} = \dots\dots\dots$
- 8) If the straight line  $\overleftrightarrow{AB} \parallel$  to x-axis, where A (8 , 3) , B (2 , k) then k =  
 $\dots\dots\dots$
- 9)  $\tan A = \frac{\sin A}{\dots\dots\dots}$
- 10) The straight line which passes through the two points (1 , y) , (3 , 4)  
its slope is  $\tan 45$  they y =  $\dots\dots\dots$
- 11)  $\cos 3x = \frac{1}{2}$  where x is an acute angle then x =  $\dots\dots\dots$
- 12) The distance between the point (2 , - 5) and the x-axis =  $\dots\dots\dots$
- 13) The equation of the straight line which passes through the point (3 , -4)  
and parallel to the x-axis is  $\dots\dots\dots$
- 14) If  $(\sqrt{2} \cos 3x = 1)$  then x =  $\dots\dots\dots$



- 15) If the point  $(0, 4)$  is the midpoint of the distance between the two points  $(-1, -1)$ ,  $(x, y)$  then the point  $(x, y)$  is .....
- 16) The slope of the straight line whose equation  $2x - 3y + 5 = 0$  equals .....
- 17) The equation of the straight line whose slope is 1 passes through the origin point is .....
- 18) DHO is isosceles triangle,  $DH = DO$ ,  $\sin D = 1$  then  $m(\angle O) = \dots\dots$
- 19) If the two straight lines  $x + y = 5$  and  $kx + 2y = 0$  are parallel, then  $k = \dots\dots\dots$
- 20) If  $X : Y$  are two complementary angles, where  $X : Y = 1 : 2$  then  $\sin X + \cos Y = \dots\dots\dots$

- (2) Find the equation of straight line passing through  $(\sqrt{3}, -2)$  and makes with the positive  $-x$  direction an angle of measure  $60^\circ$ , then calculate the length of the  $y$ -intercepted.
- (3) Find the equation of the straight line which passes through the point  $(1, 6)$  and the midpoint of  $\overline{AB}$  where  $A(1, -2)$ ,  $B(3, -4)$

(4)

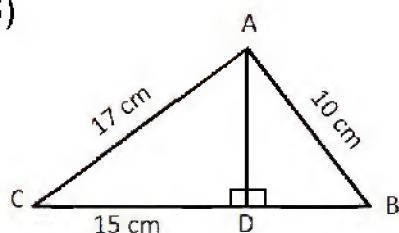


- 1) Find  $\sin X \cos Z + \cos X \sin Z$
- 2) Find  $m(\angle YXZ)$

- (5) ABCD is a parallelogram where  $A(3, 4)$ ,  $B(2, -1)$ ,  $C(-4, -3)$   
Find the coordinates of D.



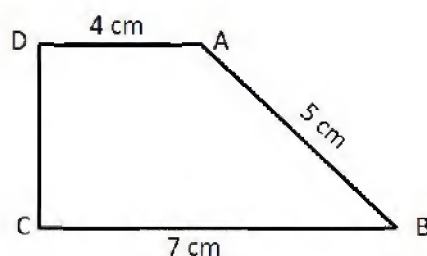
(6)

Find the value of  $3 \tan (\angle C) + \sin (\angle B)$ 

(7) If the distance between the two points  $(a, 2)$ ,  $(2a + 1, -1)$  equal 5 length unit find the value of  $a$ .

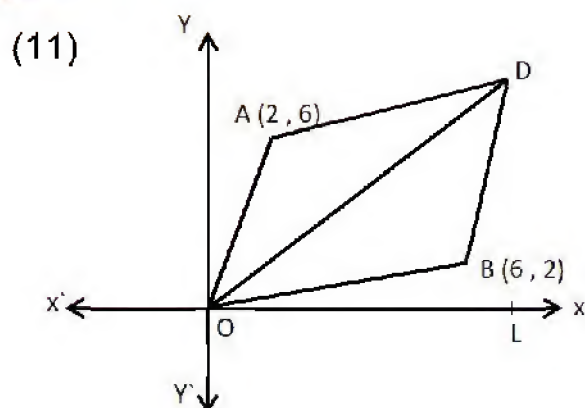
(8) Find the equation of the straight line passing through the point  $(2, -3)$  and perpendicular to the straight line  $y = 3x + 2$

(9)

Find: 1)  $\sin B$ ,  $m (\angle B)$ 

2) The surface area of trapezium ABCD

(10) ABC is a right-angled triangle at A, where  $AB = 8$  cm and  $m (\angle B) = 50^\circ$  find the length of  $\overline{AC}$  to the nearest one decimal no.



- 1) The coordinates of the point D
- 2) The equation of straight line OD
- 3)  $m(\angle DOL)$

(12) Prove that: the triangle whose vertices A (5 , -5) , B (-1 , 7) :  
C (15 , 15) is right angled at B then find its area.

(13) If the points A (-1 , -1) , B (2 , 3) and C )k , 0) are vertices of the  
right angled triangle at B

- Find: 1) The value of k  
2) The area of  $\triangle ABC$

(14) Find the slope and intercepted part of y-axis of the straight line  
whose equation  $\frac{x}{2} + \frac{y}{3} = 1$

(15) ABC is a right angled  $\triangle$  at B,  $2 AB = \sqrt{3} AC$  find the trigonometrical  
ratio of  $\angle C$

(16) Find the equation of the straight line passes through (2 , -1) and  
parallel to the straight line  $2x - y + 5 = 0$





(17)  $\overline{AB}$  is a diameter of circle M if B (8 , 11) , M (5 , 7) then find:

- 1) The coordinates of A
- 2) The length of the radius of the circle.
- 3) The equation of the perpendicular straight line to  $\overline{AB}$  from the point B.

(18) If the two equations of two straight lines  $L_1$  and  $L_2$  are  $2x - 3y + a = 0$   
,  $3x + by - 6 = 0$

- 1) Find the value of b which makes  $L_1 \parallel L_2$
- 2) Find the value of b which makes  $L_1 \perp L_2$

(19) Because of wind, the upper part of a tree was broken made an angle of measure 60 with the ground if the point of contact of the top of the tree with the ground was at distance 4 m from its bottom. Find the length of the tree to the nearest metre.



## Model Answers

### (1) Complete:

$$1) 1 \times \frac{1}{2} = \frac{1}{2}$$

$$2) -\frac{2}{1} = -2$$

$$3) \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6 - 0)^2 + (8 - 0)^2} = 10 \text{ unit}$$

$$4) m(\angle B) = 60^\circ$$

$$5) m(\angle A) = 45^\circ$$

6) The slope = 2 because the form  $y = mx + c$  and part length of y-axis = 6 unit

$$7) \text{slope of } \overleftrightarrow{CD} = \frac{2}{3}$$

8)  $\overleftrightarrow{AB} \parallel$  to x-axis it means the slope is zero

$$\text{so } \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - k}{8 - 2} = \frac{3 - k}{6} = 0$$

$$\frac{3 - k}{6} = \frac{0}{1} \rightarrow 3 - k = 0 \rightarrow -k = -3 \rightarrow k = 3$$

$$9) \tan A = \frac{\sin A}{\cos A}$$

10) The slope =  $\tan 0 = \tan 45 = 1$

$$\therefore \frac{y_2 - y_1}{x_2 - x_1} = 1 \rightarrow \frac{4 - y}{3 - 1} = 1$$

$$\frac{4 - y}{2} = \frac{1}{1}$$

$$4 - y = 2$$

$$-y = 2 - 4$$

$$-y = -2$$

$$y = 2$$

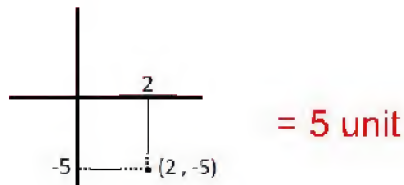


11) Shift  $\cos\left(\frac{1}{2}\right)$

$$\therefore 3x = 60$$

$$x = \frac{60}{3} = 20$$

12)



13)  $y = -4$

14)  $\cos 3x = \frac{1}{\sqrt{2}}$

$$3x = 45$$

$$x = \frac{45}{3} = 15$$

15) midpoint =  $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$(0, 4) = \left(\frac{-1+x}{2}, \frac{-1+y}{2}\right)$$

$$\frac{-1+x}{2} = \frac{0}{1}$$

$$-1 + x = 0$$

$$x = 1$$

$$\frac{-1+y}{2} = \frac{4}{1}$$

$$-1 + y = 8$$

$$y = 8 + 1 = 9$$

The point  $(x, y) = (1, 9)$

16) The slope =  $\frac{-a}{b} = \frac{-2}{3} = \frac{2}{3}$

17)  $y = mx + c$

$$y = x + c$$

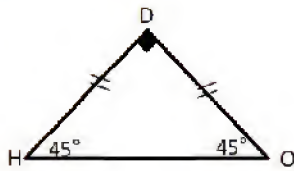
$$0 = 0 + c$$

$$c = 0$$

The final form is  $y = x$



18)



$$m(\angle O) = 45$$

because  $\sin D = 1$  so shift  $\sin(1) = 90^\circ$

$$19) \text{ Slope } 1 = \frac{-a}{b} = \frac{-1}{1} = -1$$

$$\therefore L_1 \parallel L_2$$

$$\text{Slope } 2 = \frac{-k}{2}$$

$$\text{so } \frac{-k}{2} = -1$$

$$-k = -2$$

$$k = 2$$

(20)  $m(\angle X) : m(\angle Y) : \text{sum}$

$$1 : 2 : 3$$

$$x : y : 90$$

$$m(\angle X) = \frac{1 \times 90}{3} = 30^\circ$$

$$m(\angle Y) = \frac{2 \times 90}{3} = 60^\circ$$

$$\sin 30 + \cos 60 = \frac{1}{2} + \frac{1}{2} = 1$$

(2) The slope =  $\tan 60 = \sqrt{3}$

$$\text{so } y = \sqrt{3}x + c$$

$$-2 = \sqrt{3} \times \sqrt{3} + c$$

$$-2 = 3 + c$$

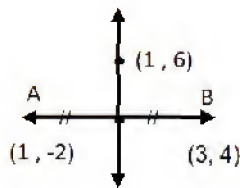
$$c = -5$$

$$\text{so } y = \sqrt{3}x - 5$$





(3)



The midpoint between A, B =  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{1+3}{2}, \frac{-2+4}{2}\right)$   
 $= (2, -3)$

So the straight line passing through (1, 6), (2, -3)

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 6}{2 - 1} = \frac{-9}{1} = -9$$

$$y = -9 + c$$

$$6 = -9 \times 1 + c$$

$$6 = -9 + c$$

$$6 + 9 = c$$

$$c = 15$$

$$y = -9x + 15$$

(4)

$$(yz)^2 = (xz)^2 - (xy)^2 = 169 - 144 = 25$$

$$yz = \sqrt{25} = 5 \text{ cm}$$

$$\text{Find: } \sin x = \frac{\text{opp.}}{\text{hyp.}} = \frac{5}{13}, \quad \cos z = \frac{\text{adj.}}{\text{hyp.}} = \frac{5}{13}$$

$$\cos x = \frac{12}{13}, \quad \sin z = \frac{12}{13}$$

$$\sin x \cos z + \cos x \sin z = \frac{5}{13} \times \frac{5}{13} + \frac{12}{13} \times \frac{12}{13} = 1$$

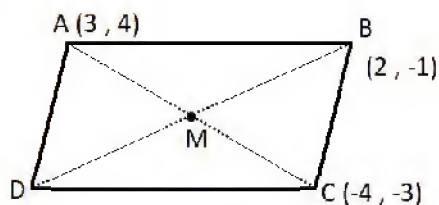
2)  $m(\angle YXZ)$

$$\therefore \sin(\angle YXZ) = \frac{5}{13}$$

$$\therefore m(\angle YXZ) = \text{shift sin}\left(\frac{5}{13}\right) = 22^\circ 37' 51''$$



(5)



$$M = \left( \frac{3-4}{2}, \frac{4-3}{2} \right)$$

$$M = \left( \frac{-1}{2}, \frac{1}{2} \right)$$

$$\left( \frac{-1}{2}, \frac{1}{2} \right) = \left( \frac{x_2+2}{2}, \frac{y_2-2}{2} \right) \rightarrow \frac{x_2+2}{2} = \frac{-1}{2} \quad \frac{y_2-2}{2} = \frac{1}{2}$$

$$x_2 = -3 \quad y_2 = 2 \quad D = (-3, 2)$$

(6)

$$(AD)^2 = (17)^2 - (15)^2 = 64$$

$$AD = \sqrt{64} = 8$$

in  $\triangle ADB$ 

$$\begin{aligned} (DB)^2 &= (AB)^2 - (AD)^2 \\ &= 100 - 64 = 36 \end{aligned}$$

$$DB = 6 \text{ cm}$$

$$3 \tan (\angle C) + \sin (\angle B) = 3 \times \frac{8}{15} + \frac{8}{10} = \frac{12}{5}$$

(7)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5 \text{ length unit}$$

$$\sqrt{(2a + 1 - a)^2 + (-1 - 2)^2} = 5$$

$$(\sqrt{(a + 1)^2 + 9})^2 = (5)^2$$

$$(a + 1)^2 + 9 = 25$$

$$(a + 1)^2 = 25 - 9$$

$$(a + 1)^2 = 16$$

$$a + 1 = \sqrt{16}$$

$$a + 1 = \pm 4$$



$$a + 1 = 4$$

$$a = 4 - 1$$

$$a = 3$$

$$a + 1 = -4$$

$$a = -4 - 1$$

$$a = -5$$

**(8)** The line passing through (2 , -3) and its slope  $-\frac{1}{3}$

$$y = mx + c$$

$$y = -\frac{1}{3}x + c$$

$$-3 = -\frac{1}{3} \times 2 + c$$

$$c = -2 + \frac{2}{3} = \frac{-7}{3}$$

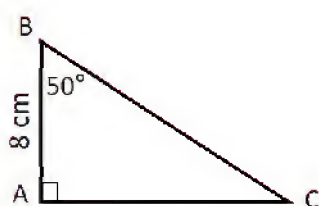
$$y = -\frac{1}{3}x + \frac{-7}{3}$$

**(9)**

$$1) \sin B = \frac{4}{5} \quad m(\angle B) = 53.1^\circ$$

$$2) \text{ the surface area} = \frac{1}{2} [b_1 + b_2] \times H = \frac{1}{2} [7 + 4] \times 4 = 22 \text{ cm}^2$$

**(10)**



$$\tan 50 = \frac{AC}{AB}$$

$$\frac{\tan 50}{1} = \frac{AC}{8}$$

$$AC = 8 \times \tan 50 = 9.5 \text{ cm}$$





(11)  $D = A + B - O$

$$1) = (2, 6) + (6, 2) - (0, 0) = (8, 8)$$

$$2) y = mx + c$$

$$\text{slope} = \frac{8-0}{8-0} = 1$$

$$y = x + c$$

$$0 = 0 + c$$

$$c = 0$$

$$\boxed{y = x}$$

$$3) \text{Slope } \overrightarrow{OD} = 1 \tan \theta$$

$$\therefore \tan \theta = 1$$

$$\therefore \theta = 45^\circ$$

(12)

$$\begin{aligned} AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 5)^2 + (7 + 5)^2} \\ &= \sqrt{36 + 144} = 6\sqrt{5} \end{aligned}$$

$$A = (5, -5), B(-1, 7)$$

$$BC = \sqrt{(15 + 1)^2 + (15 - 7)^2} = \sqrt{256 + 64} = 8\sqrt{5}$$

$$B(-1, 7), C(15, 15)$$

$$CA = \sqrt{(5 - 15)^2 + (5 - 15)^2} = \sqrt{100 + 400} = 10\sqrt{5}$$

$$(CA)^2 = (10\sqrt{5})^2 = 500$$

$$(BC)^2 + (AB)^2 = (8\sqrt{5})^2 + (6\sqrt{5})^2 = 500$$

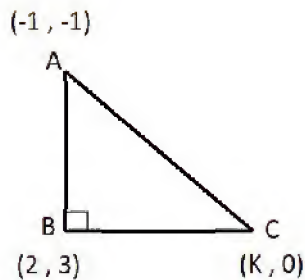
$\therefore$  ABC is right angled  $\Delta$  at B

$$\text{Area of } \Delta ABC = \frac{1}{2} (6\sqrt{5} \times 8\sqrt{5}) = 120 \text{ cm}^2$$





(13)



$$\text{Slope } AB = \frac{3 - (-1)}{2 - (-1)} = \frac{4}{3}$$

$$\text{Slope of } BC = \frac{0 - 3}{k - 2} = -\frac{3}{k - 2}$$

$$\frac{k - 2}{-3} = \frac{-4}{3}$$

$$3(k - 2) = 12$$

$$3k - 6 = 12$$

$$3k = 12 + 6$$

$$3k = 18$$

$$k = 6$$

area of  $\Delta \frac{1}{2} \times \text{base} \times \text{height}$

$$AB = \sqrt{(2 + 1)^2 + (3 + 1)^2} = 5$$

$$BC = \sqrt{(6 - 2)^2 + (0 - 3)^2} = 5$$

$$\text{area} = \frac{1}{2} \times 5 \times 5 = 12.5 \text{ cm}^2$$

(14)  $\frac{1}{2}x + \frac{1}{3}y - 1 = 0$

$$\text{Slope} = \frac{-a}{b} = \frac{-\frac{1}{2}}{\frac{1}{3}} = -\frac{1}{2} \div \frac{1}{3} = -\frac{3}{2}$$

$$\text{intercept part of y-axis} = \frac{-c}{b} = \frac{1}{\frac{1}{3}} = 3$$

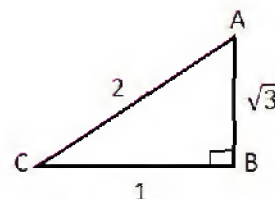
(15)  $2 AB = \sqrt{3} AC$

$$\frac{AB}{AC} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{1}{2}$$

$$\tan C = \frac{\sqrt{3}}{1}$$





**(16)** The slope of  $L_1$  = the slope of  $L_2$  because  $L_1 \parallel L_2$

$$\text{slope } L_2 = \frac{-a}{b} = \frac{-2}{-1} = 2$$

**So** Slope  $L_1 = 2$

$$y = 2x + c$$

$$2 = 2(-1) + c$$

$$2 = -2 + c$$

$$2 + 2 = c$$

$$c = 4$$

$$y = 2x + 4$$

**(17)** Midpoint =  $\left(\frac{x_1+8}{2}, \frac{y_1+11}{2}\right)$

$$(5, 7) = \left(\frac{x_1+8}{2}, \frac{y_1+11}{2}\right)$$

$1) \frac{x_1+8}{2} = 5$ $x_1 + 8 = 10$ $x_1 = 2$	$\frac{y_1+11}{2} = 7$ $y_1 + 11 = 14$ $y_1 = 3$	$A = (2, 3)$
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2) The length of radius =  $\sqrt{(8-5)^2 + (11-7)^2} = 5$  unit

3)  $\overrightarrow{AB} \perp L_1$

$$\text{Slope } \overrightarrow{AB} = \left(\frac{y_2 - y_1}{x_2 - x_1}\right) = \frac{11-3}{8-2} = \frac{8}{6} = \frac{4}{3}$$

$$\text{Slope } L_1 = -\frac{3}{4}$$

The equation  $y = mx + c$

$$y = -\frac{3}{4}x + c$$



$$11 = -\frac{3}{4} \times 8 + c$$

$$11 = -6 + c$$

$$c = 11 + 6 = 17$$

$$y = -\frac{3}{4}x + 17$$

**(18)**

1) If  $L_1 \parallel L_2$   $\therefore$  slope 1 = slope 2 =  $\frac{-a}{b}$

$$\frac{-2}{-3} = \frac{-3}{b}$$

$$\frac{2}{3} = \frac{-3}{b}$$

$$b = \frac{3(-3)}{2} = \frac{-9}{2}$$

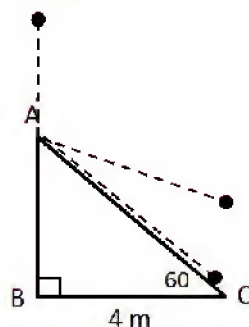
2) If  $L_1 \perp L_2$  slope 1  $\times$  slope 2 = -1

$$\text{slope 1} = \frac{2}{3}$$

$$\text{slope 2} = \frac{-3}{b}$$

$$\therefore \frac{-3}{b} = \frac{-3}{2}$$

$$b = \frac{-6}{-3} = 2$$

**(19)**

$$\frac{\tan 60}{1} = \frac{AB}{BC}$$

$$AB = 4 \tan 60$$

$$= 4\sqrt{3}$$

$$(AC)^2 = (4)^2 + (4\sqrt{3})^2 = 64$$

$$AC = 8\text{m}$$

the height of tree

$$= 4\sqrt{3} + 8 = 14.928 \approx 15\text{ m}$$



## Questions

### Part (1)

#### Lesson 1 ( Cartesian product)

- (1) If  $x = \{3, 4, 8\}$ . Find  $x \times x$  and represent it by:
- 1) arrow diagram
  - 2) Cartesian diagram
  - 3)  $n(x^2)$
- (2) If  $x = \{5\}$ ,  $y = \{2, 7\}$  find
- a)  $x \times y$  and represent it by an arrow diagram.
  - b)  $y \times x$  and represent it by Cartesian diagram.
- (3) If  $n(x) = 3$ ,  $n(x \times y) = 12$ , then  $n(y) = \dots\dots\dots$
- (4) If  $(3, 5) \in \{3, 6\} \times \{x, 8\}$ , then  $x = \dots\dots\dots$
- (5) If  $x = \{2, 5\}$ ,  $y = \{2, 4\}$ ,  $z = \{4, 6\}$ , then find:
- a)  $n(x \times (y \cup z))$
  - b)  $(x - y) \times (z \cap y)$
- (6)  $(5, 0)$  lies on  $\dots\dots\dots$
- (7)  $(-2, -3)$  lies on  $\dots\dots\dots$
- (8) a)  $x \times \emptyset = \dots\dots\dots$
- b)  $n(x \times \emptyset) = \dots\dots\dots$





## Lesson (2) Relation Function

- (1) If  $x = \{ 1, 2, 3 \}$ ,  $y = \{ 1, 3, 6, 9, 12 \}$  and  $\mathbb{R}$  is a relation from  $x$  to  $y$ , where " $a \mathbb{R} b$ " means  $a = \frac{1}{3}b$  for each  $a \in x$ ,  $b \in y$   
write  $\mathbb{R}$  and show that it is a function and write its range.
- (2) If  $x = \{ 1, 2, 3 \}$ ,  $y = \{ 2, 3, 7 \}$  and  $\mathbb{R}$  is a relation from  $x$  to  $y$ ,  
where " $a \mathbb{R} b$ " means " $a + b = \text{prime number}$ " for each  $a \in x$ ,  $b \in y$   
write  $\mathbb{R}$  and represent it by an arrow diagram.  
Is  $\mathbb{R}$  a function? Why?
- (3) If  $x = \{ 1, 3, 5 \}$ ,  $\mathbb{R}$  is a relation on  $x$ ,  $\mathbb{R} \{ (a, 3), (b, 1), (1, 5) \}$ ,  
then find the numerical value of the expression:  $a + b$
- (4) If  $x = \{ -2, 2, 5 \}$ ,  $y = \{ 3, 7, \ell \}$  and  $\mathbb{R}$  is a function from  $x$  to  $y$   
where " $a \mathbb{R} b$ " means  $b = a^2 - 1$  for each  $a \in x$ ,  $b \in y$   
a) Find the value of  $\ell$   
b) Represent  $\mathbb{R}$  by an arrow diagram



## Lesson (3) Polynomial functions

- (1) If  $f(x) = 2x - 1$ , then prove that  $f(2) - 3f(1) = \text{zero}$
- (2) If  $f$  is a function on  $x$  where  $x = \{3, 4, 5, 6\}$  and  $f(3) = 3$ ,  $f(4) = 5$ ,  $f(5) = 5$ ,  $f(6) = 5$
- Represent  $f$  by an arrow diagram.
  - Write  $f$  and mention its range.
- (3) If the function  $f = \{(0, 4), (1, 3), (2, 2), (3, 1)\}$
- Write each of domain and range of the function  $f$ .
  - Write the rule of the function  $f$ .
- (4) Represent each of the following linear functions graphically and find the point of intersection of the straight line which represents each of them with the coordinate axes, where  $x \in \mathbb{R}$
- $f : f(x) = -2x$
  - $f : f(x) = x + 2$
  - $f : f(x) = 3x$
- (5) Represent each of the following functions graphically then find the coordinates of the vertex of the curve and the equation of the line of symmetry and the maximum or minimum value of the function
- $f : f(x) = x^2 - 2x$  taking  $x \in [-2, 4]$
  - $f : f(x) = x^2 - 4x + 5$  taking  $x \in [0, 5]$
  - $f : f(x) = 1 - 3x + x^2$  taking  $x \in [-4, 4]$
- (6) If  $f(x) = 2x + b$ ,  $h(x) = b$  where  $f$  and  $h$  are polynomial functions and if  $f(1) + h(4) = 12$ , then find:  $f(4) + h(-1)$



### Lesson (4) Ratio

- (1) Find the number which if it is added to the two terms of the ratio 7 : 11 it will be 2 : 3
- (2) Find the number which if its square is added to each of the two terms of the ratio 7 : 11 it becomes 4 : 5
- (3) Two integers the ratio between them is 3 : 7 and if we subtracted 5 from each term, the ratio between each of them becomes 1 : 3, find the two numbers.





## Part (2)

### Unit [ 2 ] : Ratio, Proportion and Variation

- 1 Find the number which should be subtracted from each of the numbers 3 , 7 , 19 to be in continued proportion.

---

- 2 The ratio between two positive integers is 3 : 7. If the number 5 is subtracted from each of them, then the ratio becomes 1 : 3 find the two numbers.

---

- 3 Find the number which if it is added to the two terms of ratio 7 : 11 it will be 2 : 3

---

- 4 Two numbers the ratio between them is 2 : 3 , if you add to the first number 4 and subtract from the second number 4 the ratio will become 2 : 1 find the two numbers.

---

- 5 Find the number that if we add it to each of the numbers 1 , 7 , 25 , then they become in continued proportion.

---

- 6 If :  $3a = 2b$  , then find the value of :  $\frac{3a-b}{2a-b}$

---

- 7 If :  $\frac{x}{y} = \frac{2}{5}$  , what is the value of the expression :  $\frac{2x+y}{x+4y}$  ?

---

- 8 If :  $a, b, c$  and  $d$  are proportional quantities , prove that :  $\frac{a+2c}{b+2d} = \frac{a-c}{b-d}$

---

- 9 If :  $\frac{a}{3} = \frac{b}{2}$  Find the value of :  $\frac{a-b}{a+b}$

---

- 10 If :  $a, b, c$  and  $d$  are proportional quantities , then prove that :  $\frac{a}{b-a} = \frac{c}{d-c}$

---

- 11 If :  $a, b, c$  and  $d$  are proportional quantities , then prove that :  $\frac{a^2+c^2}{b^2+d^2} = \frac{ac}{bd}$

---

- 12 If :  $a, b, c$  and  $d$  are four real proportional quantities , then prove that :  $\frac{ac}{bd} = \left( \frac{a-c}{b-d} \right)^2$

---

- 13 If :  $\frac{a}{3} = \frac{b}{5}$  , then find the value of :  $\frac{7a+9b}{4a+2b}$

---

- 14 If :  $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$  , then prove that :  $\frac{a-b+c}{a+b-c} = \frac{1}{3}$





15 If :  $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3x}$  Find value of :  $x$

16 If :  $\frac{a}{4} = \frac{b}{5} = \frac{c}{3}$  then find the value of :  $\frac{a-b+c}{a+b-c}$

17 If :  $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$  , then prove that :  $\frac{2y-z}{3x-2y+z} = \frac{1}{2}$

18 If :  $\frac{21x+a}{7x+b} = \frac{a}{b}$  and  $x \neq 0$  , then find the value of :  $\frac{a+2b}{2a}$

19 If :  $\frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$  Prove that :  $\frac{x+y+z}{x-z} = 5$

20 If :  $\frac{x+y}{5} = \frac{y+z}{3} = \frac{z+x}{6}$  Prove that :  $\frac{x-z}{2} = \frac{x+y+z}{7}$

21 If :  $a, b, c$  and  $d$  are in continued proportion , prove that :  $\frac{ab-cd}{b^2-c^2} = \frac{a+c}{b}$

22 If  $y$  is the middle proportional between  $x$  and  $z$  , prove that :  $\frac{xz}{y(y+z)} = \frac{x}{x+y}$

23 If :  $1, x, 9, y$  are continued proportion where  $x$  and  $y$  are positive numbers, then Find each of  $x$  and  $y$

24 If :  $b$  is a middle proportional between  $a$  and  $c$  , then prove that :  $\frac{b}{b+c} = \frac{a-b}{a-c}$

25 If :  $b$  is a middle proportional of  $a$  and  $c$  , prove that :  $\frac{b}{b+c} = \frac{a}{a+b}$

26 If :  $b$  is the middle proportional between  $a, c$  , prove that :  $\frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$

27 If :  $b$  is the middle proportion between  $a$  and  $c$  , prove that :  $\frac{2c^2-3b^2}{2b^2-3a^2} = \frac{c}{a}$

28 If :  $x, y$  and  $z$  are proportional prove that :  $\frac{x^2+y^2}{y^2+z^2} = \frac{x}{z}$

29 If :  $\frac{a^2+b^2}{b^2} = \frac{b^2+c^2}{c^2}$  , then prove that :  $b$  is the middle proportional between  $a$  and  $c$



30 If  $y$  varies as  $X$  and  $y = \frac{5}{3}$ , when  $X = \frac{1}{6}$  Find the value of  $X$  when  $y = \frac{3}{4}$

---

31 If :  $y \propto X$  and  $y = 20$  when  $X = 7$ , find  $y$  when  $X = 14$

---

32 If :  $y \propto X$  and  $y = 15$  when  $X = 3$

Find the relation between  $X$  and  $y$ , then find  $y$  when  $X = 7$

---

33 If :  $y \propto X$  and  $y = 1$  when  $X = 4$

(1) Find the relation between  $X$  and  $y$

(2) Find the value of  $X$  when  $y = 8$

---

34 If :  $y \propto X$ , then  $y = 14$  when  $X = 42$ , then find :

(1) The relation between  $X$  and  $y$

(2) The value of  $y$  when  $X = 60$

---

35 If :  $y$  inverse variation as  $X$  and  $y = 6$  at  $X = 2$ , then find :

(1) The relation between  $y$ ,  $X$

(2)  $y$  at  $X = 3$

---

36  $y \propto \frac{1}{X}$  and  $y = 3$  when  $X = 2$

Find : (1) The relation between  $X$  and  $y$

(2) The value of  $y$  when  $X = 1.5$

---

37 If  $y$  changes inversely with  $X$  and  $y = 2$  when  $X = 4$

(1) Find the relation between  $X$  and  $y$

(2) Find the value of  $y$  when  $X = 16$

---

38 If :  $y \propto \frac{1}{X}$ ,  $X = 4$  when  $y = 9$ , then find the relation between  $X$  and  $y$

and find the value of  $X$  when  $y = 24$

---

39 If :  $X = z + 8$ ,  $z$  varies inversely as  $y$  and  $z = 2$ , when  $y = 3$

Find :  $y$  when  $X = 3$

---

40 If :  $y = 1 + b$  where  $b$  varies inversely as the square of  $X$ , and  $y = 17$  when  $X = \frac{1}{2}$

Find the relation between  $y$  and  $X$  then find the value of  $y$  when  $X = 2$

---

41 If :  $a^2 b^2 + \frac{1}{4} = ab$ , then prove that :  $a$  varies inversely as  $b$

---





- 42 If :  $4x^2 + 9y^2 = 12xy$  , then prove that :  $x$  varies as  $y$
- 
- 43 If :  $y = b - 5$  and  $y \propto x$  and  $b = 19$  when  $x = 2$  , then find the relation between :  $y$  and  $x$
- 
- 44 If :  $y = 3 + a$  , and  $a \propto \frac{1}{x}$  and  $y = 5$  when  $x = 1$  Find :  
(1) The relation between  $y$  and  $x$   
(2) The value of  $y$  when  $x = 2$
- 
- 45 A car moves with a uniform velocity where the covered distance varies directly with the time. If the car covered 150 km in 6 hours, find the distance covered by that car in 10 hours.
- 
- 46 If the weight of a body on the earth ( $w$ ) varies as its weight on the moon ( $r$ )  
If  $w_1 = 182$  kg when  $r_1 = 35$  kg , find  $r_2$  when  $w_2 = 312$  kg.
- 
- 47 If the velocity ( $v$ ) of the water running through a pipe varies inversely as the square of the radius of the pipe ( $r$ ) and  $v = 5$  cm/sec, when  $r = 3$  cm.  
Find :  $v$  when  $r = 15\frac{3}{4}$  cm.



**Unit [ 3 ] : Statistics**

- 1 Calculate the arithmetic mean and the standard deviation of the set of values :  
16 , 32 , 5 , 20 and 27

- 2 Calculate the standard deviation for the values : 3 , 12 , 17 , 28 , 30

- 3 Calculate the mean and standard deviation to the following data :  
12 , 13 , 16 , 18 and 21

- 4 Calculate the arithmetic mean and the standard deviation of the set of values 73 ,  
54 , 62 , 71 , 60

- [ 5 ] The following table represents the number of children of 100 families in a city

Number of children	0	1	2	3	4	Total
Number of families	6	15	40	25	14	100

Calculate each of the arithmetic mean and standard deviation.

- [ 6 ] The following table represents the number of children of 26 families in a city :

Number of children	0	1	2	3	4	5	Total
Number of families	9	1	6	3	5	2	26

Calculate the standard deviation.

- [ 7 ] Find the arithmetic mean and standard deviation of the following data :

The set	0 -	2 -	4 -	6 -	8 -
The frequency	5	9	15	15	6





# Model Answers

## Part (1)

### Lesson (1) Cartesian product

$$(1) x \times x = \{ (3, 3), (3, 4), (3, 8), (4, 3), (4, 4), (4, 8), (8, 3), (8, 4), (8, 8) \}$$

$$n(x^2) = 9$$

Draw it by yourself

$$(2) x \times y = \{ (5, 2), (5, 7) \}$$

$$y \times x = \{ (2, 5), (7, 5) \}$$

Draw it by yourself

$$(3) n(y) = 4$$

$$(4) x = 5$$

$$(5) a) 6 \quad b) \{ (5, 4) \}$$

$$(6) x\text{-axis}$$

$$(7) 3^{\text{rd}} \text{ quad.}$$

$$(8) a) \emptyset \quad b) \text{zero}$$

### Lesson (2) relations

$$(1) \mathbb{R} = \{ (1, 3), (2, 6), (3, 9) \} \quad , \text{yes it's a function}$$

$$\text{Range} = \{ 3, 6, 9 \}$$

$$(2) \mathbb{R} = \{ (1, 2), (2, 3), (3, 2) \} \quad ,$$

yes because each element of  $x$  has one and only one image of  $y$

$$(3) a + b = 8$$

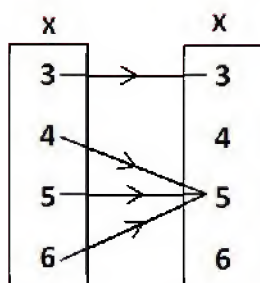
$$(4) \ell = 24$$



### Lesson (3) Polynomial Functions

(1)  $f(2) = 3, f(1) = 1 \rightarrow f(2) - 3f(1) = 3 - 3 \times 1 = \text{zero}$

(2)



$$f = \{ (3, 3), (4, 5), (5, 5), (6, 5) \}$$

$$\text{Range} = \{ 3, 5 \}$$

(3)  $\text{domain} = \{ 0, 1, 2, 3 \}$

$$\text{Range} = \{ 4, 3, 2, 1 \}$$

$$\text{Rule } b = 4 - a$$

(4) a)  $f(1) = -2$                        $(1, -2)$

$$f(0) = 0$$
                       $(0, 0)$

$$f(-1) = 2$$
                       $(-1, 2)$

b)

x	-1	0	1
f(x)	1	2	3

c)

x	-1	0	1
f(x)	1	2	3

Draw it by yourself

(5) Do it yourself

(6)  $f(1) = 2 \times 1 + b$

$$h(4) = b$$

$$2 + b + b = 12 \rightarrow 2b = 10 \rightarrow b = 5$$

$$f(x) = 2x + 5, \quad h(x) = 5$$

$$f(4) = 8 + 5 = 13 \rightarrow f(4) + h(-1) = 13 + 5 = 18$$



## Lesson (4) Ratio

$$(1) \frac{x+7}{x+11} = \frac{2}{3} \rightarrow 3x + 21 = 2x + 22$$
$$\rightarrow 3x - 2x = 22 - 21 \rightarrow x = 1$$

$$(2) \frac{x^2+7}{x^2+11} = \frac{4}{5} \rightarrow 5x^2 + 35 = 4x^2 + 44$$
$$\rightarrow x^2 = 9 \rightarrow x = \pm \sqrt{9} = \pm 3$$

(3) Let the two numbers = x and y

$$\frac{x-5}{y-5} = \frac{1}{3}, \frac{x}{y} \rightarrow x = 3m, y = 7m$$

$$\frac{3m-5}{7m-5} = \frac{1}{3} \rightarrow 9m - 15 = 7m - 5 \rightarrow 2m = 10 \rightarrow m = 5$$

$$x = 3 \times 5 = 15, y = 7 \times 5 = 35$$





## Part (2)

### Unit [ 2 ] : Answers

- (1) let the number be  $X$ :

$$\therefore \frac{3-X}{7-X} = \frac{7-X}{19-X}$$

$$\therefore (7-X)^2 = (3-X)(19-X)$$

$$\therefore X^2 - 14X + 49 = X^2 - 22X + 57$$

$$\therefore 8X = 8 \quad \therefore X = 1$$

$\therefore$  The number = 1

- (2) Let the two numbers be  $3X$  ;  $7X$

$$\therefore \frac{3X-5}{7X-5} = \frac{1}{3} \quad \therefore 9X-15 = 7X-5$$

$$\therefore 2X = 10 \quad \therefore X = 5$$

$\therefore$  The two numbers are : 15 and 35

- (3) Let the number be  $X$   $\therefore \frac{7+X}{11+X} = \frac{2}{5}$

$$\therefore 21 + 3X = 22 + 2X \quad \therefore X = 1$$

$\therefore$  The number is : 1

- (4) let the numbers be  $2X$  and  $3X$

$$\therefore \frac{2X+4}{3X-4} = \frac{2}{1}$$

$$\therefore 2X+4 = 6X-8$$

$$\therefore 4X = 12$$

$$\therefore X = 3$$

$\therefore$  The two numbers are : 6 and 9

- (5) Let the number be  $X$ :

$$\therefore \frac{1+X}{7+X} = \frac{7+X}{25-X}$$

$$\therefore (7+X)^2 = (1+X)(25+X)$$

$$\therefore X^2 + 14X + 49 = X^2 + 26X + 25$$

$$\therefore 12X = 24$$

$$\therefore X = 2$$

$\therefore$  The number is 2

$$(6) \therefore 3a = 2b \quad \therefore \frac{a}{b} = \frac{2}{3}$$

$$\therefore a = 2m, b = 3m$$

$$\therefore \frac{3a-b}{2a-b} = \frac{3(2m)-3m}{2(2m)-3m} = \frac{6m-3m}{4m-3m} = \frac{3m}{m} = 3$$

$$(7) \therefore \frac{X}{y} = \frac{2}{5} \quad \therefore X = 2m, y = 5m$$

$$\therefore \frac{2X+y}{X+4y} = \frac{4m+5m}{2m+20m} = \frac{9m}{22m} = \frac{9}{22}$$

$$(8) \therefore \frac{a}{b} = \frac{c}{d} = m \quad \therefore a = bm, c = dm$$

$$\therefore \frac{a+2c}{b+2d} = \frac{bm+2dm}{b+2d} = \frac{m(b+2d)}{b+2d} = m \quad (1)$$

$$\therefore \frac{a-c}{b-d} = \frac{bm-dm}{b-d} = \frac{m(b-d)}{b-d} = m \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{a+2c}{b+2d} = \frac{a-c}{b-d}$$

$$(9) \therefore \frac{a}{3} = \frac{b}{2} = m \quad \therefore a = 3m, b = 2m$$

$$\therefore \frac{a-b}{a+b} = \frac{3m-2m}{3m+2m} = \frac{m}{5m} = \frac{1}{5}$$

$$(10) \therefore \frac{a}{b} = \frac{c}{d} = m \quad \therefore a = bm, c = dm$$

$$\therefore \frac{a}{b-a} = \frac{bm}{b-bm} = \frac{bm}{b(1-m)} = \frac{m}{1-m} \quad (1)$$

$$\therefore \frac{c}{d-c} = \frac{dm}{d-dm} = \frac{dm}{d(1-m)} = \frac{m}{1-m} \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{a}{b-a} = \frac{c}{d-c}$$

$$(11) \therefore \frac{a}{b} = \frac{c}{d} = m \quad \therefore a = bm, c = dm$$

$$\therefore \frac{a^2+c^2}{b^2+d^2} = \frac{b^2m^2+d^2m^2}{b^2+d^2} = \frac{m^2(b^2+d^2)}{b^2+d^2} = m^2 \quad (1)$$

$$\therefore \frac{ac}{bd} = \frac{bm \times dm}{bd} = \frac{bdm^2}{bd} = m^2 \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{a^2+c^2}{b^2+d^2} = \frac{ac}{bd}$$

$$(12) \therefore \frac{a}{b} = \frac{c}{d} = m \quad \therefore a = bm, c = dm$$

$$\therefore \frac{ac}{bd} = \frac{bm \times dm}{bd} = \frac{bdm^2}{bd} = m^2 \quad (1)$$

$$\therefore \left(\frac{a-c}{b-d}\right)^2 = \left(\frac{bm-dm}{b-d}\right)^2 = \left(\frac{m(b-d)}{b-d}\right)^2 = m^2 \quad (2)$$

$$\text{From (1) and (2) : } \therefore \frac{ac}{bd} = \left(\frac{a-c}{b-d}\right)^2$$

$$(13) \therefore \frac{a}{3} = \frac{b}{5} = m \quad \therefore a = 3m, b = 5m$$

$$\therefore \frac{7a+9b}{4a+2b} = \frac{7 \times 3m + 9 \times 5m}{4 \times 3m + 2 \times 5m} = \frac{21m + 45m}{12m + 10m}$$





$$= \frac{66m}{22m} = 3$$

$$(14) \therefore \frac{a}{4} = \frac{b}{5} = \frac{c}{3} = m \text{ where } m > 0$$

$$\therefore a = 4m, b = 5m, c = 3m$$

$$\therefore \text{L.H.S.} = \frac{a-b+c}{a+b-c} = \frac{4m-5m+3m}{4m+5m-3m} = \frac{2m}{6m} = \frac{1}{3} = \text{R.H.S.}$$

$$(15) \therefore \frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-b+5c}{3X}$$

• multiplying the two terms of the 1<sup>st</sup> ratio by 2 and the 2<sup>nd</sup> by -1 and the 3<sup>rd</sup> by 5 and adding the antecedents and consequents of the three ratios.

$$\therefore \frac{2a-b+5c}{4-3+20} = \text{one of the given ratios.}$$

$$\therefore \frac{2a-b+5c}{21} = \frac{2a-b+5c}{3X} \therefore 3X = 21 \therefore X = 7$$

$$(16) \therefore \frac{a}{4} = \frac{b}{5} = \frac{c}{3} = m \text{ where } m > 0$$

$$\therefore a = 4m, b = 5m, c = 3m$$

$$\therefore \frac{a-b+c}{a+b-c} = \frac{4m-5m+3m}{4m+5m-3m} = \frac{2m}{6m} = \frac{1}{3}$$

$$(17) \therefore \frac{x}{3} = \frac{y}{4} = \frac{z}{5} = m \text{ where } m > 0$$

$$\therefore x = 3m, y = 4m, z = 5m$$

$$\therefore \text{L.H.S.} = \frac{2y-z}{3x-2y+z} = \frac{8m-5m}{9m-8m+5m} = \frac{3m}{6m} = \frac{1}{2} = \text{R.H.S.}$$

$$(18) \therefore \frac{21x+a}{7x+b} = \frac{a}{b} \therefore 21xb + ab = 7xa + ba$$

$$\therefore 21xb = 7xa \therefore \frac{a}{b} = \frac{21x}{7x} = 3$$

$$\therefore a = 3m, b = m$$

$$\therefore \frac{a+2b}{2a} = \frac{3m+2m}{6m} = \frac{5m}{6m} = \frac{5}{6}$$

$$(19) \therefore \frac{x+y}{7} = \frac{y+z}{5} = \frac{z+x}{8}$$

• adding the antecedents and consequents of the three ratios.

$$\therefore \frac{x+y+y+z+z+x}{7+5+8} = \frac{2(x+y+z)}{20} = \frac{x+y+z}{10}$$

= one of the given ratios. (1)

• multiplying the terms of the 2<sup>nd</sup> ratio by -1 and adding the antecedents and consequents of 1<sup>st</sup> and 2<sup>nd</sup> ratios.

$$(20) \therefore \frac{x+y}{5} = \frac{y+z}{3} = \frac{z+x}{6}$$

• subtracting the antecedent and consequent of the 2<sup>nd</sup> ratio from the antecedent and consequent of the 1<sup>st</sup> ratio

$$\therefore \frac{x+y-y-z}{5-3} = \frac{x-z}{2} = \text{one of the given ratios (1)}$$

• adding the antecedents and consequents of the three ratios

$$\therefore \frac{x+y+y+z+z+x}{5+3+6} = \frac{2x+2y+2z}{14} = \frac{x+y+z}{7} = \text{one of the given ratios (2)}$$

$$\text{From (1) and (2): } \therefore \frac{x-z}{2} = \frac{x+y+z}{7}$$

$$(21) \therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = m$$

$$\therefore c = dm, b = dm^2, a = dm^3$$

$$\therefore \frac{a-b-cd}{b^2-c^2} = \frac{dm^3 \times dm^2 - dm^2 \times d}{d^2m^4 - d^2m^2} = \frac{d^2m^5 - d^2m}{d^2m^4 - d^2m^2} = \frac{d^2m(m^4-1)}{d^2m^2(m^2-1)} = \frac{(m^2-1)(m^2+1)}{m(m^2-1)} = \frac{m^2+1}{m} \quad (1)$$

$$\therefore \frac{a+c}{b} = \frac{dm^3+dm}{dm^2} = \frac{dm(m^2+1)}{dm^2} = \frac{m^2+1}{m} \quad (2)$$

$$\text{From (1) and (2): } \therefore \frac{a-b-cd}{b^2-c^2} = \frac{a+c}{b}$$

$$(22) \therefore \frac{x}{y} = \frac{y}{z} = m \therefore y = zm, x = zm^2$$

$$\therefore \frac{xz}{y(y+z)} = \frac{zm^2 \times z}{zm(zm+z)} = \frac{z^2m^2}{z^2m(m+1)} = \frac{m}{m+1} \quad (1)$$

$$\therefore \frac{x}{x+y} = \frac{zm^2}{zm^2+zm} = \frac{zm^2}{zm(m+1)} = \frac{m}{m+1} \quad (2)$$

$$\text{From (1) and (2): } \therefore \frac{xz}{y(y+z)} = \frac{x}{x+y}$$

$$(23) \therefore \frac{1}{x} = \frac{x}{9} = \frac{9}{y} \therefore x = \sqrt{9 \times 1} = 3, y = \frac{9 \times 9}{3} = 27$$

$$(24) \therefore \frac{a}{b} = \frac{b}{c} = m \therefore b = cm, a = cm^2$$

$$\therefore \frac{b}{b+c} = \frac{cm}{cm+c} = \frac{cm}{c(m+1)} = \frac{m}{m+1} \quad (1)$$

$$\therefore \frac{a-b}{a-c} = \frac{cm^2-cm}{cm^2-c} = \frac{cm(m-1)}{c(m^2-1)} = \frac{m(m-1)}{(m+1)(m-1)} = \frac{m}{m+1} \quad (2)$$



$$= \frac{m(m-1)}{(m+1)(m-1)} = \frac{m}{m+1} \quad (2)$$

From (1) and (2) :  $\therefore \frac{b}{b+c} = \frac{a-b}{a-c}$

$$(25) \therefore \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{b}{b+c} = \frac{cm}{cm+c} = \frac{cm}{c(m+1)} = \frac{m}{m+1} \quad (1)$$

$$\therefore \frac{a}{a+b} = \frac{cm^2}{cm^2+cm} = \frac{cm^2}{cm(m+1)} = \frac{m}{m+1} \quad (2)$$

from (1) and (2) :  $\therefore \frac{b}{b+c} = \frac{a}{a+b}$

$$(26) \therefore \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{c^2 m^4}{c^2 m^2} + \frac{c^2 m^2}{c^2} = m^2 + m^2 = 2m^2 \quad (1)$$

$$\therefore \frac{2a}{c} = \frac{2cm^2}{c} = 2m^2 \quad (2)$$

From (1) and (2) :  $\therefore \frac{a^2}{b^2} + \frac{b^2}{c^2} = \frac{2a}{c}$

$$(27) \therefore \frac{a}{b} = \frac{b}{c} = m \quad \therefore b = cm, a = cm^2$$

$$\therefore \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{2c^2 - 3c^2 m^2}{2c^2 m^2 - 3c^2 m^4} = \frac{c^2(2-3m^2)}{c^2 m^2(2-3m^2)} = \frac{1}{m^2} \quad (1)$$

$$\therefore \frac{c}{a} = \frac{c}{cm^2} = \frac{1}{m^2} \quad (2)$$

From (1) and (2) :  $\therefore \frac{2c^2 - 3b^2}{2b^2 - 3a^2} = \frac{c}{a}$

$$(28) \therefore \frac{x}{y} = \frac{y}{z} = m \quad \therefore y = zm, x = zm^2$$

$$\therefore \frac{x^2 + y^2}{y^2 + z^2} = \frac{z^2 m^4 + z^2 m^2}{z^2 m^2 + z^2} = \frac{z^2 m^2 (m^2 + 1)}{z^2 (m^2 + 1)} = m^2 \quad (1)$$

$$\therefore \frac{x}{z} = \frac{zm^2}{z} = m^2 \quad (2)$$

From (1) and (2) :  $\therefore \frac{x^2 + y^2}{y^2 + z^2} = \frac{x}{z}$

$$(29) \therefore \frac{a^2 + b^2}{b^2} = \frac{b^2 + c^2}{c^2}$$

$$\therefore a^2 c^2 + b^2 c^2 = b^4 + b^2 c^2$$

$$\therefore b^4 = a^2 c^2 \quad \therefore b^2 = ac$$

$\therefore b$  is the middle proportional between  $a$  and  $c$

$$(30) \therefore y \propto x \quad \therefore \frac{y_1}{y_2} = \frac{x_1}{x_2} \quad \therefore \frac{\frac{5}{3}}{\frac{3}{4}} = \frac{1}{x_2}$$

$$\therefore x_2 = \frac{\frac{3}{4} \times \frac{1}{6}}{\frac{5}{3}} = \frac{3}{40}$$

$$(31) \therefore y \propto x \quad \therefore y = mx$$

$$\therefore 20 = 7m \quad \therefore m = \frac{20}{7}$$

$$\therefore y = \frac{20}{7} x \quad \therefore y = \frac{20}{7} \times 14 = 40$$

$$(32) \therefore y \propto x \quad \therefore y = mx \quad \therefore 15 = 3m$$

$$\therefore m = 5 \quad \therefore y = 5x \text{ At } x = 7 \quad \therefore y = 5 \times 7 = 35$$

$$(33) (1) \therefore y \propto x \quad \therefore y = mx \quad \therefore 1 = 4m$$

$$\therefore m = \frac{1}{4} \quad \therefore y = \frac{1}{4} x$$

$$(2) \text{ At } y = 8 \quad \therefore 8 = \frac{1}{4} x \quad \therefore x = 32$$

$$(34) (1) \therefore y \propto x \quad \therefore y = mx \quad \therefore 14 = 42m$$

$$\therefore m = \frac{1}{3} \quad \therefore y = \frac{1}{3} x$$

$$(2) \text{ At } x = 60 \quad \therefore y = \frac{1}{3} \times 60 \quad \therefore y = 20$$

$$(35) (1) \therefore y \propto \frac{1}{x} \quad \therefore xy = m$$

$$\therefore 2 \times 6 = m \quad \therefore m = 12$$

$$(2) \text{ At } x = 3 \quad \therefore 3 \times y = 12 \quad \therefore y = 4$$

$$(36) (1) \therefore y \propto \frac{1}{x} \quad \therefore xy = m \quad \therefore 3 \times 2 = m$$

$$\therefore m = 6 \quad \therefore xy = 6$$

$$(2) \text{ At } x = 1.5 \quad \therefore 1.5y = 6 \quad \therefore y = 4$$

$$(37) \therefore y \propto \frac{1}{x} \quad \therefore xy = m \quad \therefore 4 \times 2 = m$$

$$\therefore m = 8 \quad \therefore xy = 8$$

$$\text{At } x = 16 \quad \therefore 16y = 8 \quad \therefore y = \frac{1}{2}$$

$$(38) \therefore y \propto \frac{1}{x} \quad \therefore xy = m \quad \therefore 4 \times 9 = m$$

$$\therefore m = 36 \quad \therefore xy = 36$$

$$\text{At } y = 24 \quad \therefore 24x = 36 \quad \therefore x = 1.5$$

$$(39) \therefore z \propto \frac{1}{y} \quad \therefore yz = m \quad \therefore 3 \times 2 = m$$

$$\therefore m = 6 \quad \therefore z = \frac{6}{y} \quad \therefore x = \frac{6}{y} + 8$$

$$\text{At } x = 3 : \therefore 3 = \frac{6}{y} + 8 \quad \therefore -5 = \frac{6}{y}$$

$$\therefore y = -\frac{6}{5}$$





$$\begin{aligned} (40) \because b &\propto \frac{1}{x^2} & \therefore b &= \frac{m}{x^2} \\ \therefore y &= 1 + \frac{m}{x^2} & \therefore 17 &= 1 + \frac{m}{(\frac{1}{2})^2} \\ \therefore 16 &= \frac{m}{\frac{1}{4}} & \therefore m &= 4 \\ \therefore y &= 1 + \frac{4}{x^2} \\ \text{At } x &= 2 : \therefore y &= 1 + \frac{4}{4} & \therefore y = 2 \end{aligned}$$

$$\begin{aligned} (41) \because a^2 b^2 - ab + \frac{1}{4} &= 0 \\ \therefore \left(ab - \frac{1}{2}\right)^2 &= 0 & \therefore ab - \frac{1}{2} &= 0 \\ \therefore ab &= \frac{1}{2} & \therefore a &\propto \frac{1}{b} \end{aligned}$$

$$\begin{aligned} (42) \because 4x^2 - 12xy + 9y^2 &= 0 \\ \therefore (2x - 3y)^2 &= 0 & \therefore 2x - 3y &= 0 \\ \therefore 2x &= 3y & \therefore x &= \frac{3}{2}y \\ \therefore x &\propto y \end{aligned}$$

$$\begin{aligned} (43) \because y &\propto x & \therefore y &= mx & \therefore b - 5 &= mx \\ \therefore 19 - 5 &= 2m & \therefore 14 &= 2m & \therefore m &= 7 \\ \therefore y &= 7x \end{aligned}$$

$$\begin{aligned} (44) (1) \because a &\propto \frac{1}{x} & \therefore a &= \frac{m}{x} & \therefore y &= 3 + \frac{m}{x} \\ \therefore 5 &= 3 + \frac{m}{1} & \therefore m &= 2 & \therefore y &= 3 + \frac{2}{x} \\ (2) \text{ At } x &= 2 & \therefore y &= 3 + \frac{2}{2} & \therefore y &= 4 \end{aligned}$$

$$\begin{aligned} (45) \because d &\propto t & \therefore \frac{d_1}{d_2} &= \frac{t_1}{t_2} & \therefore \frac{150}{d_2} &= \frac{6}{10} \\ \therefore d_2 &= \frac{150 \times 10}{6} & &= 250 \text{ km.} \end{aligned}$$

$$\begin{aligned} (46) \because w &\propto r & \therefore \frac{w_1}{w_2} &= \frac{r_1}{r_2} \\ \therefore \frac{182}{312} &= \frac{35}{r_2} & \therefore r_2 &= \frac{312 \times 35}{182} = 60 \text{ kg.} \end{aligned}$$

$$\begin{aligned} (47) \because v &\propto \frac{1}{r^2} & \therefore \frac{v_1}{v_2} &= \frac{r_2^2}{r_1^2} \\ \therefore \frac{5}{v_2} &= \frac{\left(15\frac{3}{4}\right)^2}{(3^2)} & \therefore \frac{5}{v_2} &= \frac{248.0625}{9} \\ \therefore v_2 &= \frac{5 \times 9}{248.0625} = \frac{80}{441} \text{ cm/sec.} \end{aligned}$$

## Unit [ 3 ] : Answers

- (1) Form the tables by yourself ; then :  
The mean  $\bar{X} = 20$  ;  $\sigma = 9.32$
- (2) Form the tables by yourself ; then :  $\sigma = 10.06$
- (3) The mean  $\bar{X} = \frac{12 + 13 + 16 + 18 + 21}{5} = 16$   
; form the tables by yourself ; then :  $\sigma = 3.29$
- (4) The mean  $= \frac{73 + 54 + 62 + 71 + 60}{5} = 64$   
; Form the tables by yourself  
; then  $\sigma = 7.07$
- (5) Form the tables by yourself  
; then the mean  $\bar{X} = 2.26$  ;  $\sigma = 1.06$
- (6) Form the tables by yourself ; then :  $\sigma = 1.73$
- (7) Form the tables by yourself  
; then the mean  $\bar{X} = 5.32$  ;  $\sigma = 2.31$